Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
B		2121743

Abstract

In 2020, a severe wildfire broke out in Australia, which has seriously endangered the safety of the people and the global climate. Therefore, the Australian fire extinguishing department will take measures to detect the fire through drones and frontline personnel to extinguish the fire.

And the nonlinear optimization model adopted in this paper has a great effect on the allocation of two different equipment UAVs. The adopted analytic hierarchy process grey forecasting model has played a supervisory role in the assessment of Australian wildfires in the next ten years.

This thesis mainly needs to study three issues. The first is to optimize the combination of two different types of drones. The second problem is to estimate the future outbreak of wildfires and the factors that affect economic expenditure based on the form and conditions of the first year. For the third question, we need to optimize the space on the basis of the first question to make the solution of the problem more comprehensive. In response to the three problems, we adopted nonlinear forecasting models, grey forecasting models, and combined optimization models.

After calculation, we concluded that a total of 45 drones should be purchased. In order to improve the search sensitivity and ensure the effective supply of power, 9 radio drones should be selected, and all the remaining quotas should be drones equipped with repeaters. In the next ten years, the cost of making drones will increase mainly due to environmental pollution.

Keywords:Grey prediction model;Portfolio Optimization;Nonlinear optimization

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Contents

1 Introduction

1.1 Problem Restatement

In this paper, we wished to estimate the optimal number of Surveillance and Situational Awareness (SSA) drones and radio repeater drones for the Victorian Country Fire Authority (CFA) to purchase. We gave predictions as to how our prediction should also include a general method to optimally locate the repeater drones for use in the field. Our model incorporated safety concerns for the firefighters, and the changing climate.

1.2 Background

Frontline wildfire firefighters use handheld two-way radios to send status updates to and receive orders from the Emergency Operations Center (EOC). However, these radios have low ranges; 5km in open areas and 2km in areas with large deviations in topography. The short range of handheld communications limits the effective range of firefighting operations since the frontline firefighters must be only a few kilometers from the EOC in order to receive status updates. This puts a limit on how much firefighters can actually accomplish. Of course, the frontline firefighters could move outside the range of the handheld radios, but they would be missing important updates and orders from the EOC, and would not be able to utilize the SSA drones. It is for this reason the radio repeater drones are used.

A fire can travel at a speed of 5km/h over flat ground and doubles in speed for every 10° of uphill slope (The Bushfire Foundation). The potential for a rapidly changing situation means that firefighters must have constant access to information, especially in areas with complicated topographies. The radio repeater drones can extend the firefighters operational range, while keeping them in contact with the EOC. This contact is important because the EOC receives information from the SSA drones about the evolving situation. The frontline teams are thus tethered to the EOC.

We also considered the changing climate's effect on wildfires in the near foreseeable future. The intensity of wildfires is exacerbated by drought conditions; as the amount of dry fuel increases (Williams, 2013). In our changing climate, drought conditions will become more frequent and more intense, putting strain on the CFA's fire fighting abilities. For this reason, we should expect rising equipment costs in the organization over time.

There are other considerations we must consider with this model. For instance, the drones have limited battery life and definite charging times. Additionally, their on-board devices (video/telemetry or radio repeaters) have their own power consumptions and effective ranges. Our model incorporates these factors as well, especially in considering the operation of these equipment.

1.3 Related Data

From the problem statement, we have several pieces of information about each of the aspects in this question. Regarding the ranges of the communication equipment:

- Handheld radios used by frontline firefighters and the EOC have a range 5km in open area and 2km in areas with obstructions
- The radio repeater drones have a range of 20km, independent of topography due to the flight height of drones

- The SSA drones must be within the range of frontline firefighters handheld communications to receive telemetry data from handheld sensors.

These data constitute the main constraints of the problem, for the operation of communication equipment is impossible if these constraints are ignored.

In optimizing the communications system, we should consider the power usage of the system. Thus, we are given the power consumption for these devices:

- Handheld devices use 5W of electrical power.
- The radio repeater and video/telemetry devices use 10W of power.

Finally, we have the specification for the WileE-15.2X drone:

- The range is 30km and is determined by the distance at which it can be controlled.
- The maximum speed of the drone is 20m/s (approximately 72km/h)
- The maximum flight time is 2.5 hours and the recharge time is 1.75 hours
- Each drone costs $10000 (AUD) and does not depend on what equipment is attached.

The drone specifications provide key constraints on the scale of possible firefighter operations. The limited flight range imposes an upper bound on the total area the SSA drones can monitor and the radio repeater drones can provide communications service. The maximum flight speed puts a limit on the number of specific actions a drone can perform in a given time. The charge/discharge cycle means we must utilize back-up drones in our plan, to ensure continuing operation while drones are recharging. The cost for each drone is high and will be the weightiest trade-off for buying new drones.

1.4 Problem Summary

1. To monitor the fire with the highest efficiency, the most economical and the safest as the goal, group the radio drone and repeater drone
2. For the purpose of maintaining the stability and sensitivity of the model, predict the extreme situations and the increase economic costs
3. Determine a model for optimizing the locations of VHF/VHU

2 Analysis of the Problem

This question is based on the range of the Australian wildfire burning, the location of the Victorian terrain, the number of drones and repeaters to establish an optimal combination of monitoring and predicting future wildfires based on conditions that exacerbate wildfire situations.

For problem 1, We need to ensure that the allocated drones can maintain the maximum range of communication and minimize the cost and power consumption while ensuring that they are familiar with the terrain.

For problem 2, we have to build a prediction model depending on the present limited data. From question 1, we will have an overview of the number of drones we assigned in this situation. And have basic information about controlling this wildfire.Based on the fact that we only have current data and there is no forecast for future wildfires, we have to establish a grey forecast model and calculate the form of the next ten years through iterative methods.

For question 3, we will come back to the question1. In the first question there is already a basic control of the number of devices and the range of the wildfire. In this way we will create a model combined with plane distribution.

3 Modelling Assumptions

1. All data are real data, or extrapolated from actual data by mathematics process, the source is real
2. Use the communication area as the distance traveled by frontline personnel
3. Except for the impact of terrain and fire, the same machine has the same working ability

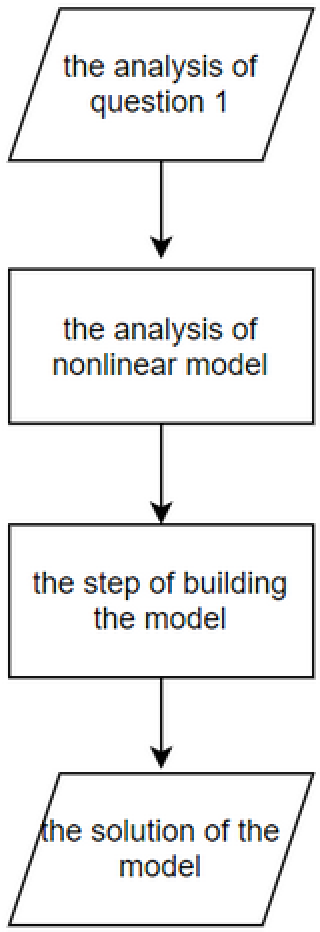
4 Symbols Explanations

Table 1: Symbols and Their Meaning

Symbol	Meaning
λ	Transfer Coefficient
ρ	Grey Prediction Model Coefficient
α	Model Accuracy Factor
v	Vertex
D	Optimization Model for Feasible Region
f	Objective Function

5 Model Building and Solving

5.1 Question 1 Analysis



For the first question, we actually solved the combined optimal solution of the radio drones and the drones equipped with a repeater in consideration of the actual terrain, equipment conditions and organizational rules.The question also requires us to find the optimal combination of the number of radio drones and drones equipped with repeaters while balancing capabilities, safety and economy and taking into account the distribution of communications and geographic locations.Through evaluation and analysis of the frequency and coefficient of wildfire spread, combined with geographical location, it is convenient to estimate the total number and optimal combination of drones required and it is convenient to establish a wildfire tracking evaluation system in the future to predict the trends. Due to the excessive spread of wildfires, it is difficult to accurately calculate the specific arrangement of drones with front lines.

5.1.1 The analysis of the nonlinear programming model

We use nonlinear programming to build an optimization model.Non-linear optimization is an optimization problem that is called a non-linear programming problem if at least one of the objective function or constraint conditions is a nonlinear function. And the nonlinear optimization model allows users to have a clear idea to consider optimization goals and limiting factors, and obtain accurate results through optimization tools. In this question, since we are considering the optimal number of two types of drones equipped with different combinations, we also need to consider safety factors, the influence of geographical environment factors on information dissemination, the spread of wildfires, and the economy. We choose to use a nonlinear optimization model to deal with optimization problems in a variety of complex situations. Non-linear optimization models require objective functions and constraints.

5.1.2 Model Building

The first step is to convert actual problems into mathematical problems. Since the prices of the two different drone configurations are the same, we can monitor more scopes by positioning the optimized target within a certain period of time. Minimum power consumption while ensuring high efficiency.

Depending on the mathematical problem that has been simplified, find the decision conditions that constitute the limiting objective function, and list them.By analyzing the terrain of Australia's wildfire area, we can get a rough estimate that 2/3of the range of frontline firefighters is rural and 1/3 is urban. And then we could ex

5.1.3 Model solving

Assume x_1 to be the number of SSA drones, which have a 2km range in the city and 5km range in the country and assume x_2 is the number of drones with repeaters. According to the background materials, we have

$$x_1 + x_2 = C,$$

where C is a positive integer.

Since the SSA drones operate differently in the city versus the country, we assign the number Y_1 to the country and Y_2 to the city, and we have the special case $Y_1 = 2/3$. Assume Y is a function of the monitor range. Then,

$$Y = \frac{2}{3}x_1\left(\pi r_1^2\right) + \frac{1}{3}x_1\left(\pi r_2^2\right) + x_2\left(\pi r^2\right)$$

where $r_1 = 5\text{km}$, $r_2 = 2\text{km}$, and $r = 20\text{km}$. Then, we have

$$Y = 18\pi x_1 + 400\pi x_2.$$

Let E be the total energy cost of the drones. Since the power usage for the SSA drones is $P_1 = 5\text{W}$ and for the repeater drones, $P_2 = 10\text{W}$ we can calculate the total power usage over the course of the times $t_1 = 2.5\text{h} * 3600\text{s/h}$ and $t_2 = 1.75\text{h} * 3600\text{s/h}$;

$$E = P_1 t_1 x_1 + P_2 t_2 x_2 = 45000 x_1 + 63000 x_2.$$

Finally, the cost of the drones is simple to compute

$$S = (x_1 + x_2) * 10000$$

We wish to maximize Y while minimizing the trade-off E, keeping the number of drones equal to C. We assume the total cost function is

$$J = -Y + E$$

And we have

$$J = (45000 - 18\pi)x_1 + (63000 - 400\pi)x_2$$

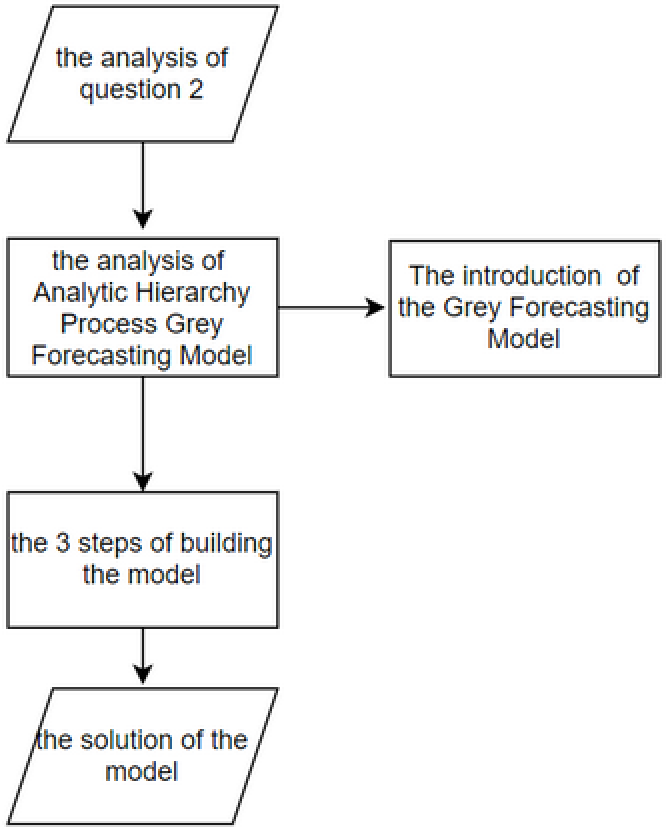
or equivalently

$$J = -16800 x_1 + 61743 C.$$

We wish to minimize the cost function J (thus maximizing the range function Y). That gives us an inspiration that the less number of SSA is good for us to save the cost of J. The extreme case is $x_1 = 0$; we employ all the drones with repeaters and in this cases $x_2 = C$. So,

$$\min(J) = 61743 * C$$

5.2 Question 2 Analysis



5.2.1 The analysis of analytic hierarchy model

For the second question, we will use the analytic hierarchy process to predict. Analytic Hierarchy Process (AHP) is a systematic and hierarchical analysis method that combines qualitative and quantitative analysis. The characteristic of this method is that on the basis of in-depth research on the nature, influencing factors and internal relationships of complex decision-making problems, it uses less quantitative information to mathematize the decision-making thinking process, thereby providing multi-objective, multi-criteria or Complex decision-making problems with no structural characteristics provide simple decision-making methods. It is a model and method for making decisions on complex systems that are difficult to fully quantify.

5.2.2 Model Building

Assume we make n observations for a quantity X_0 such that

$$X_0(k) = X_0(1) + X_0(2) + \ldots + X_0(n)$$

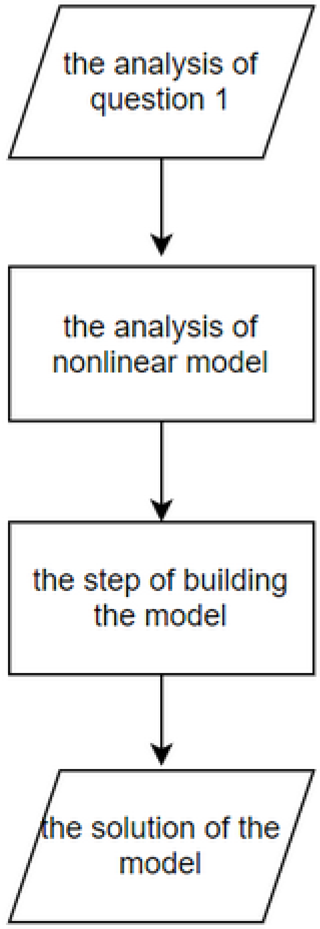
so that we get the equation

$$\eta_0 = \frac{\text{min min} X_0(k) + \hat{x}(k) + \text{max max} \rho X_0(k) + \hat{x}(k)}{X_0(k) + \hat{x}(k) + \text{max max} \rho X_0(k) + \hat{x}(k)}.$$

We then find the gray number of development $\alpha = \alpha/\mu$ where μ is the Internal gray number Finally, we solve the following differential equation to get the grey prediction model,

$$\hat{x}(k) = \left(X(1) - \frac{\alpha}{\mu} \right) e^{-\alpha k} + \frac{\mu}{\alpha}, \qquad k = 0, 1, 2, ..., n$$

5.3 Question 3 Analysis



5.3.1 The Analysis of Question 3

Question 3 is mainly based on the optimization of the space plane on the basis of question 1. We do not consider the spatial distribution of drones, but only want to get the highest efficiency arrangement of drones.

5.3.2 Modelling Analysis

- Through the map, we can know a few more serious wildfire locations. We consider that these drones basically pass through every city.
- We build the objective function f and assuming take off from a certain point to set the best path
- Find the feasible region of the objective function D and get the optimization matrix

6 Error Analysis

6.1 Error Analysis of Question 1

- The assessment of rural or urban distribution in Australia's wildfire area may be inaccurate
- May be inaccurate for the Australian government's total budget C
- We did not consider the organization and arrangement of the aircraft during actual flight

6.2 Error Analysis of Question 2

- Only a single variable is considered when solving the gray prediction model
- Since the model itself is iterative prediction based on a small amount of data, it cannot accurately predict the data several years later

6.3 Error Analysis of Question 3

- Without reasonable consideration of the coordination and scheduling of different types of drones
- The considered feasible region is too large and set to the entire Victoria area
- If the number of drones purchased in practice is not very large, the solution obtained by the optimal matrix set by the model will be biased

7 Modelling Evaluation and Promotion

7.1 Modelling Advantages

The advantage of this model is that under limited conditions, we can use these three models to get a basic solution.

7.2 Modeling Disadvantages

And it can also predict the direction of the next few years through the predictive model

7.3 Modeling Promotion

We want to start with accurate data, we can carefully set the route to pre-evaluate the range of drone driving, and then conduct investigations to find a reasonable budget, so that the feasible domain constraints in the optimization model will increase, and the data obtained will be more accurate.

8 Strengths and weaknesses

8.1 Strengths

- **Applies widely**
  This system can be used for many types of airplanes, and it also solves the interference during the procedure of the boarding airplane,as described above we can get to the optimization boarding time.We also know that all the service is automate.
- **Improve the quality of the airport service**
  Balancing the cost of the cost and the benefit, it will bring in more convenient for airport and passengers.It also saves many human resources for the airline.

9 Annotated Budget Request

Dear Victoria Country Fire Authority,

We divided fees of the project into three different kinds, one is from the drones directly, followed by the second is from the station built for service, and the other one is about the fees of personals. We assume the three kinds of stations according to the hazard zones of the wildfire, one is the most dangerous zone, which is named as the front line, and the second one is the less dangerous zone, which is named as the middle, and the last one is in service for backup, which is in safety zone.

According to our analysis earlier, we can assign the numbers of SSA and Drones with repeater (Named as Repeater for convenience) as 5 and 45 respectively. And among them, approximately we have 2 of 5 of the SSA as the reserved ones; 5 of 45 of the Drones with repeaters as reserved ones. So the drones number will be listed as Table.2

Table 2: List of the number of drones

Kinds of Drone	Number
SSA in Duty	3
SSA Reserved	2
Repeater in Duty	40
Repeater Reserved	5

The total const of the drones would then be

$$C_{\text{drones}} = [(3+2)+(40+5)] * \$10000 = \$500000$$

That will be the first cost of fighting wildfires with drones.

We assign 3 different kinds of station to serve the Drones and crew members, the front line, the middle intensified one and the safety zone one.

Tab.3 is listed about the distribution of the number of the crew members of fire fight workers and drones.

Table 3: List of the number of the drones, crew members and the station

Kinds of Station	Number of Drones plus Crew Member
Front station in Duty	6(2+1+2+1)+12
Middle station	40(1+1+35+3)+80
Safety zone station	4(0+0+3+1)+8

Notes of Tab.2, for the 6(2+1+2+1)+12, 6 is the total number of the drones, and according to the sequential order from left to the right in the (), they are SSA of duty, SSA of reserved, repeater in duty and repeater of reserved. And 12 is the number of crew members. And we can guess the same meaning from the middle station and the safety zone station.

The first one is the front of the wildfire, it includes all the 2 drones in duty of SSA and 1 drone SSA reserved; same quantities of Repeaters are configured also with them, so, we also have 3 repeaters in the first station, and 1 one them is reserved. And the number of firefighters to take care of drones and in charge of communication are double of the number of the drones, that is 12 of them. And we can have the second extra cost of Fighting Wildfires with Drones:

$$C_{\text{Wildfire front}} = 12 * \$1000 + \$5000 = \$17000$$

Note: the second extra fee of \$5000 is for the fee of the station. So,

$$C_{\text{Wildfire middle}} = 80 * \$1000 + \$5000 = \$85000$$

$$C_{\text{Wildfire safety}} = 80 * \$1000 + \$5000 = \$13000$$

Please note that we assume the hand held devices and the small sized apparatuses fees are included in the fees of crew members.

Sincerely,
Team 2121743

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Appendices

Appendix A Problem 1 Code

Input matlab source:

```matlab
% This is the pareto.m of optimization of the following problem;
% min {mu1=x1^2+4x2,mu2=x2^2+2};
% x1,x2
% sbject to
% 2x1+3x2^2-8<=0;
%x1+x2-7/2=0;
%0<=x1<=10;
%0<=x2<=5;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Initial Guess;
x0=[1 1];
%Lower and upper bounds;
LB=[0 0];
UB=[10 5];
%Initialize counter j;
j=1;
% for loop that changes the weights.
% Each iteration of the for loop produces one Pareto point;
for i=0:0.02:1
     w1=1;
     w2=1-i;
     [x,fval]=fmincon(@objun,x0,[],[],[],[],LB,UB,@confun,[],w1,w2);
     solx(j)=x(1)^2+4*x(2);
     soly(j)=x(2)^2+2;
     j=j+1;
end

%Ploting the Pareto frontier;
plot(solx,soly,'*');
xlabel('objective 1');
ylabel('objective 2');



function[c,ceq]=confun(x,w1,w2)
c(1)=[2*x(1)+3*x(2)^2-8];
ceq=[x(1)+x(2)-7/2];
end
```

```
function [f]=objun(x,w1,w2)
f1=x(1)^2+4*x(2);
f2=x(2)^2+2;
f=w1*f1+w2*f2;
end
```