ECE 433 Power Systems Analysis - Lab 1

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11/15

In this lab, a four-bus system was used to perform a power flow study by hand, with PowerWorld, and with Matlab Matpower to compare and contrast the results. The four-bus system has the given data:

Sbase = 100MVA, Vbase = 230kV

Bus Data:

Bus	Pg	Qg	Pd	Qd	V	Type
1			50	30.99	$1.00\angle 0$	Slack
2	0	0	170	105.35	$1.00\angle 0$	PQ
3	0	0	200	123.94	$1.00\angle 0$	PQ
4	350		80	49.58	$1.02\angle 0$	PV

Branch Data:

	Series Z	Series Z	Shunt Y
From-To	R p.u	X p.u.	Bp.u.
1-2	0.01008	0.0504	0.1025
1-3	0.00744	0.0372	0.0775
2-4	0.00744	0.0372	0.0775
3-4	0.01272	0.0636	0.1275

Part 1:

In part one of this lab, the slack bus (bus 1) power and reactive power were found by using one iteration of the Gauss-Seidel power flow solution method. To do so, first the 4-bus system was drawn for use in the calculation, with transmission line shunt admittance split in half, with one half at each bus end of the transmission lines as is common practice.

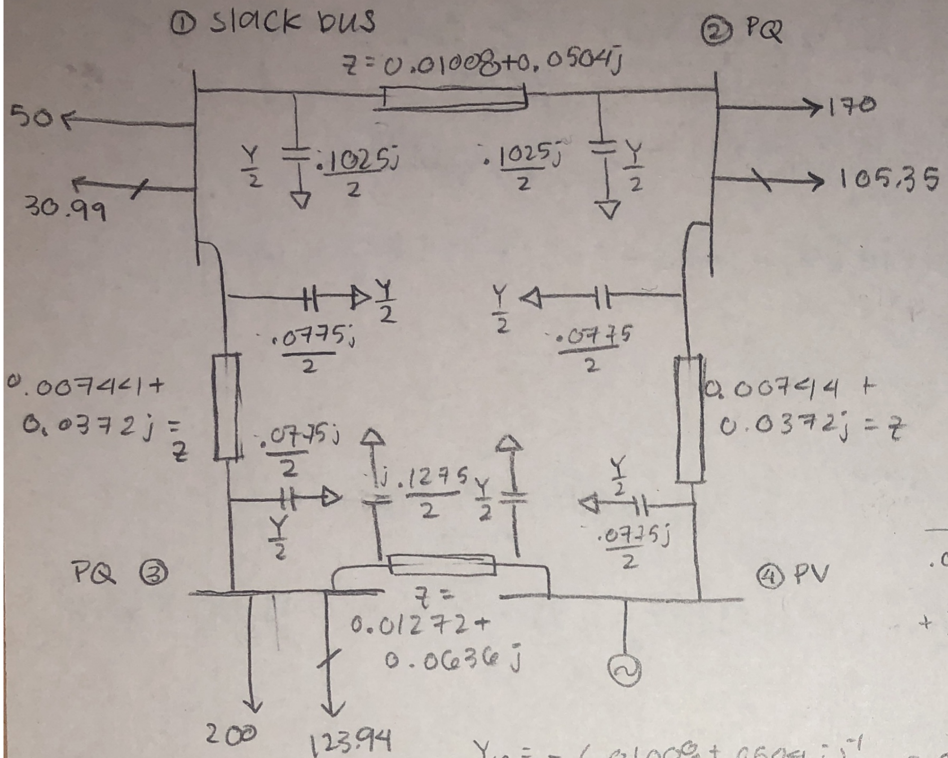
Next, the Y bus matrix elements were found. Ykk elements are found by calculating the summation of all impedances directly connected to bus k. Ykn elements are found by calculating the summation of all admittances directly connecting bus k to bus n, and negating this result. The calculated Y bus matrix can be seen in the hand calculations on the next page.

After finding the Y bus, the unknowns for bus 2, 3, and 4 were found, to be used in bus 1's power and reactive power calculations. One of the major difficulties of calculating power flows by hand that I have found, with either the Gauss-Seidel or Newton-Raphson method, is the impact of rounding. On these calculations, with oftentimes small numbers, every time a value is rounded, the impact is great. The work done to perform this calculation can be seen on the next two pages. I tried to round to at least 4 decimal places.

step 1 draw:

$S_{base} = 100\,MVA$

$V_{base} = 230\,kV$



find P & Q @ slack bus w/ GS method (one iteration).

step 2
find Ybus: $\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$

$Y_{11} =$

$$\frac{1}{.01008+.0504j} + \frac{1}{.00744+.0372j} + j\frac{.1025}{2} + j\frac{.0775}{2}$$

$$= 8.9852 - 44.8360$$

$Y_{12} = -(.01008+.0504j)^{-1} = -3.8156 + 19.0781j = Y_{21}$

$Y_{13} = -(.007441+.0372j)^{-1} = -5.1696 + 25.8478j = Y_{31}$, $Y_{14} = 0 = Y_{41}$ $\quad Y_{23} = 0 = Y_{32}$

$Y_{22} = (.01008+.0504j)^{-1} + (.00744+.0372j)^{-1} + \frac{.1025j}{2} + \frac{.0775j}{2} = 8.9595 - 44.8479j$

$Y_{24} = -(.00744+.0372j)^{-1} = -5.1696 + 25.8478j = Y_{42}$

$Y_{33} = \frac{(.0775j)}{2} + (.00744+.0372j)^{-1} + (.01272+.0636j)^{-1} + \frac{(.1275j)}{2} = 8.1933 - 40.8638j$

$Y_{34} = -(.01272+.0636j)^{-1} = -3.0237 + 15.1186j = Y_{43}$

$$Y_{44} = \frac{.0775j}{2} + \frac{.1275}{2}j + \frac{1}{.01272+.0636j} + \frac{1}{.00744+.0372j}$$

$8.1933 - 40.86384j$

step 3 calculate:

- for bus 2, find V_K and δ_K
- for bus 3, find V_K and δ_K
- for bus 4, find Q_K and δ_K

→ during flat start, $V_K \angle \delta_K = 1 \angle 0°$

then use $P_1 = V_1 \sum_{n=1}^{4} Y_{1n} V_n \cos(\delta_K - \delta_n - \theta_{Kn})$

$Q_1 = V_1 \sum_{n=1}^{4} Y_{1n} V_n \sin(\delta_K - \delta_n - \theta_{Kn})$

for bus 1, P_1 and Q_1, @ $i=0$ for iteration 0

$P_1 = (1) \sum_{n=1}^{4} Y_{1n} V_n \cos(0 - \delta_n - \theta_{Kn})$ $\quad Q_1 = (1) \sum_{n=1}^{4} Y_{1n} V_n \sin(0 - \delta_n - \theta_{Kn})$

Ybus:

$$\begin{bmatrix} 8.99-44.85j & -3.81+19.08j & -5.17+25.85j & 0 \\ -3.81+19.08j & 8.99-44.85j & 0 & -5.17+25.85 \\ -5.17+25.85j & 0 & 8.19-40.86j & -3.02+15.12j \\ 0 & -5.17+25.85j & -3.02+15.12j & 8.12-40.86j \end{bmatrix}$$

$$V_2: \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*(0)} - \sum_{n=1}^{1} Y_{2n}V_n(i+1) - \sum_{n=2+1}^{4} Y_{2n}V_n(i)\right] = V_2(i+1)$$

$$V_2(0+1) = \frac{1}{Y_{22}}\left[\frac{-1.7 + 1.0535j}{1\angle 0} - Y_{21}\overbrace{V_1(1)}^{1\angle 0} - \cancel{Y_{23}V_3(0)}^{\,Y_{23}=0} - Y_{24}\overbrace{V_4(0)}^{1.02\angle 0}\right]$$

$$= .9835621 - .0323158j$$, plug back in for $V_2^*(0)$

<u>$V_2(1)$, corrected</u> $= .9841600674 - .0338118841j \approx$ <u>$.9847\angle -1.9677°$</u>

note: $P_{GK} = P_K + P_{DK}$, $Q_{GK} = Q_K + Q_{DK}$

$$V_3: \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*(0)} - \sum_{n=1}^{2} Y_{3n}V_n(1) - \sum_{n=4}^{4} Y_{3n}V_n(0)\right] = V_3(1)$$

$$V_3(1) = \frac{1}{Y_{33}}\left[\frac{-2 + 1,2394j}{1\angle 0} - Y_{31}\overbrace{V_1(1)}^{1\angle 0} - \cancel{Y_{32}V_2(1)}^{\,Y_{32}=0} - Y_{34}\overbrace{V_4(0)}^{1,02\angle 0}\right] = .971218 - .041692j$$

<u>$V_3(1)$, corrected</u> $= .9719657006 - .0445380684j \approx$ <u>$.9730\angle -2.6236°$</u>

$$V_4: \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^*(0)} - \sum_{n=1}^{3} Y_{4n}V_n(1)\right] = V_4(1)$$

$$Q_4: \operatorname{Im}\left\{V_4(0)\left[\sum_{n=1}^{4} Y_{4n}V_n(0)\right]^*\right\} = \operatorname{Im}\left\{1,02\left[Y_{42} + Y_{43} + Y_{44}(1,02)\right]^*\right\}$$

$$= \operatorname{Im}\{.163866 + .714776j\} = \underline{.714776}$$

$$V_4(1) = \frac{1}{Y_{44}}\left[\frac{2,7 + .218976j}{1\angle 0} - \cancel{Y_{41}\overbrace{V_1(1)}^{1\angle 0}}^{\,0} - Y_{42}\overbrace{V_2(1)}^{\text{solved for above}} - Y_{43}\overbrace{V_3(1)}^{\text{solved for above}}\right]$$

$\approx .989588 + .026217j$, <u>$V_4(1)$, corrected</u> $\approx .9913899 + .0266479j$

$\approx$ <u>$.9917\angle 1.5397°$</u>

$$P_1 + jQ_1 = V_1\left[\sum_{n=1}^{N} Y_{kn}V_n\right]^* \qquad Y_{14} = 0$$

$$= 1\left[Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + \overbrace{Y_{14}}^{}V_4\right]^*$$

$= 2.001631192 + .577654395j$ per units, $S_{base} = 100$ MVA

$P_1 \approx 200.16$ MW , $Q_1 \approx 57.77$ MVAr

Part 2:

Part two of this lab uses PowerWorld to analyze the same 4-bus system. The process is much simpler, with the system being drawn in the program, then the 'solve power flow' function being used. For this lab, both the Gauss-Seidel and Newton-Raphson functions were used on this system. Figure 2.1 shows the system as entered in PowerWorld.

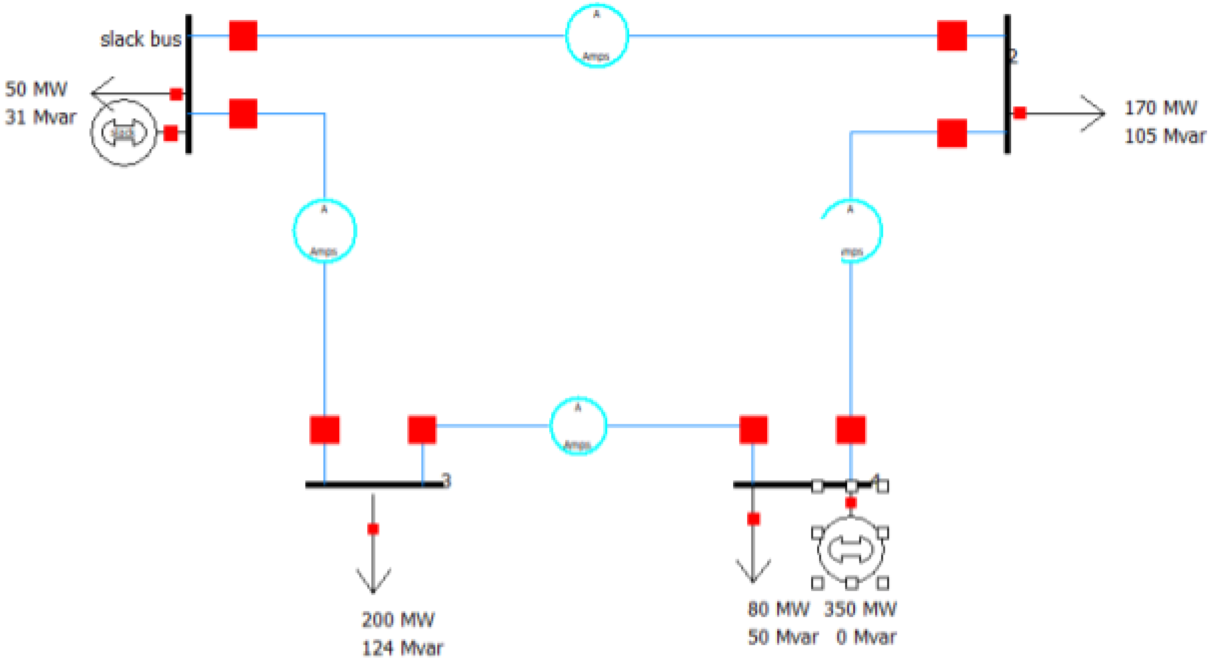
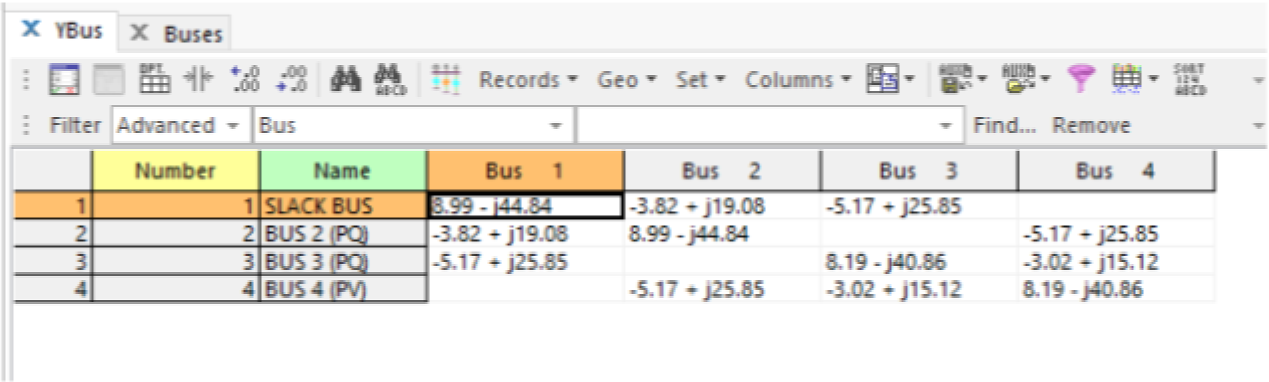
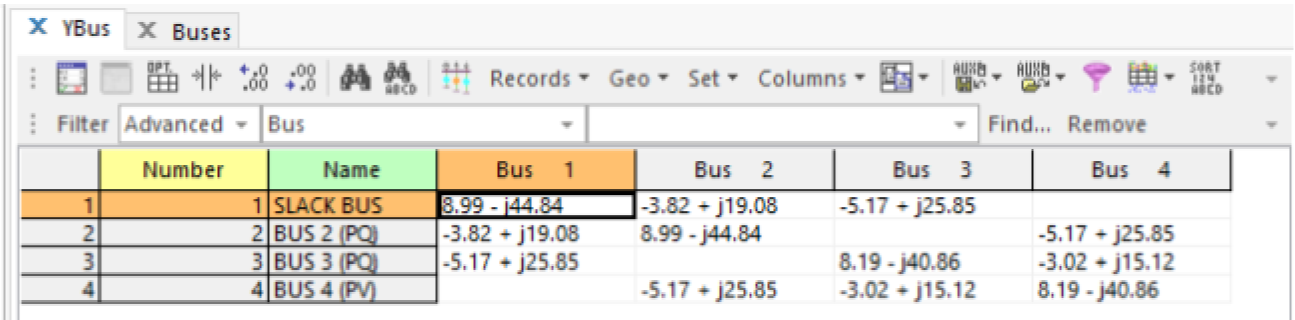


Figure 2.1

For each method, the Ybus matrices were the same. The software-generated Ybus matrices also had the same entry values as my calculated Ybus entries from Part 1 (at least, when rounded to two decimal points as the program does). The comparison can be seen in Figure 2.2.



	Number	Name	Bus 1	Bus 2	Bus 3	Bus 4
1	1	SLACK BUS	8.99 - j44.84	-3.82 + j19.08	-5.17 + j25.85	
2	2	BUS 2 (PQ)	-3.82 + j19.08	8.99 - j44.84		-5.17 + j25.85
3	3	BUS 3 (PQ)	-5.17 + j25.85		8.19 - j40.86	-3.02 + j15.12
4	4	BUS 4 (PV)		-5.17 + j25.85	-3.02 + j15.12	8.19 - j40.86

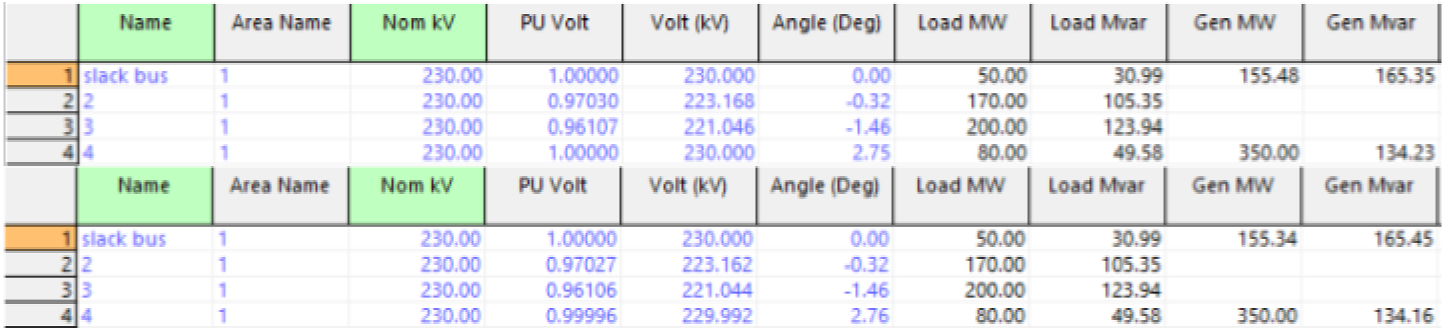


	Number	Name	Bus 1	Bus 2	Bus 3	Bus 4
1	1	SLACK BUS	8.99 - j44.84	-3.82 + j19.08	-5.17 + j25.85	
2	2	BUS 2 (PQ)	-3.82 + j19.08	8.99 - j44.84		-5.17 + j25.85
3	3	BUS 3 (PQ)	-5.17 + j25.85		8.19 - j40.86	-3.02 + j15.12
4	4	BUS 4 (PV)		-5.17 + j25.85	-3.02 + j15.12	8.19 - j40.86

Figure 2.2

The Ybus for Newton-Raphson (top) and Gauss-Seidel (bottom) solutions in PowerWorld are identical, and match the hand-calculated Ybus as well. The secondary diagonal of the matrix is all populated with zeros.

The PU voltage results between the Newton-Raphson and Gauss-Seidel methods, however, are slightly different. The voltage magnitudes are the same to three decimal places. The angles are mostly the same, but only include two decimal places in the result, so it is not clear if they are rounded to the same value or if the algorithms calculated the exact same angles. I think, the angles probably round to the same values. It is not surprising that the results are not identical, as the methods utilize different mathematical principles to approximate the unknown values. The resulting PU voltage magnitudes and angles from PowerWorld can be seen in figure 2.3 on the next page. Additionally, generator and branch data from both Gauss-Seidel and Newton-Raphson methods of solving with PowerWorld are included in figure 2.4.

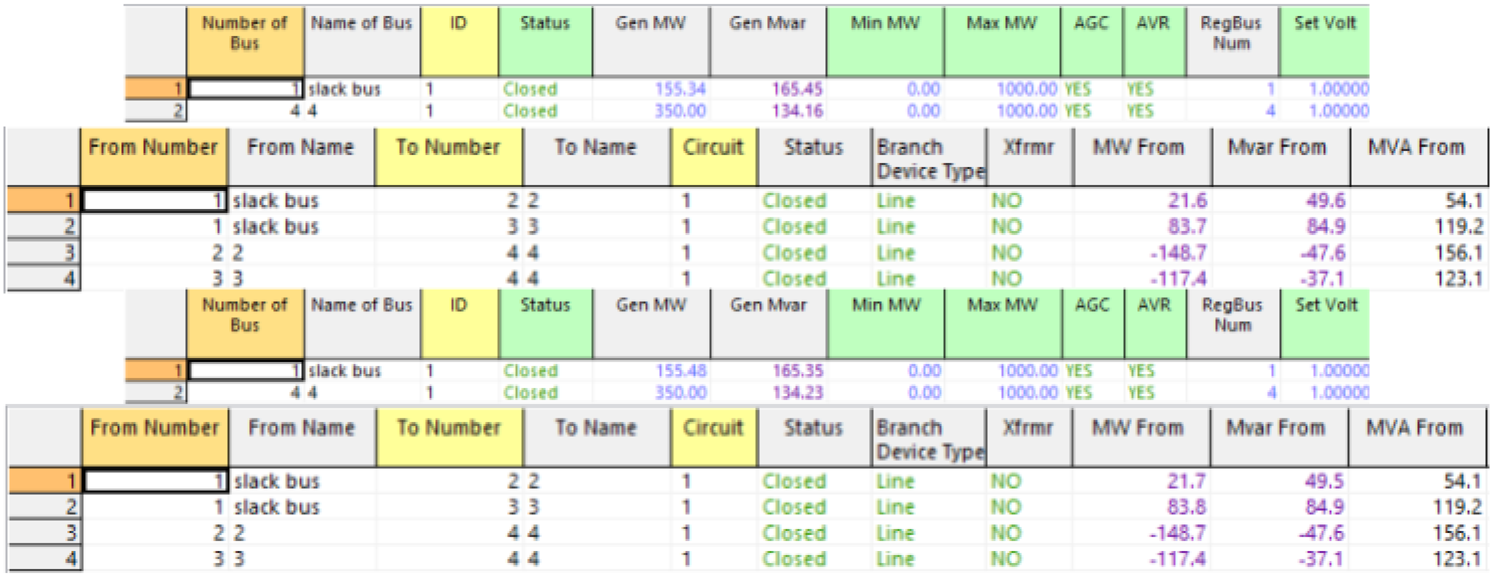


	Name	Area Name	Nom kV	PU Volt	Volt (kV)	Angle (Deg)	Load MW	Load Mvar	Gen MW	Gen Mvar
1	slack bus	1	230.00	1.00000	230.000	0.00	50.00	30.99	155.48	165.35
2	2	1	230.00	0.97030	223.168	-0.32	170.00	105.35		
3	3	1	230.00	0.96107	221.046	-1.46	200.00	123.94		
4	4	1	230.00	1.00000	230.000	2.75	80.00	49.58	350.00	134.23

	Name	Area Name	Nom kV	PU Volt	Volt (kV)	Angle (Deg)	Load MW	Load Mvar	Gen MW	Gen Mvar
1	slack bus	1	230.00	1.00000	230.000	0.00	50.00	30.99	155.34	165.45
2	2	1	230.00	0.97027	223.162	-0.32	170.00	105.35		
3	3	1	230.00	0.96106	221.044	-1.46	200.00	123.94		
4	4	1	230.00	0.99996	229.992	2.76	80.00	49.58	350.00	134.16

Figure 2.3

This is a screen shot of the Gauss Seidel PU voltages and angles from PowerWorld (top) and the Newton Raphson PU voltages from PowerWorld (bottom).



	Number of Bus	Name of Bus	ID	Status	Gen MW	Gen Mvar	Min MW	Max MW	AGC	AVR	RegBus Num	Set Volt
1	1	slack bus	1	Closed	155.34	165.45	0.00	1000.00	YES	YES	1	1.00000
2	4	4	1	Closed	350.00	134.16	0.00	1000.00	YES	YES	4	1.00000

	From Number	From Name	To Number	To Name	Circuit	Status	Branch Device Type	Xfrmr	MW From	Mvar From	MVA From
1	1	slack bus	2	2	1	Closed	Line	NO	21.6	49.6	54.1
2	1	slack bus	3	3	1	Closed	Line	NO	83.7	84.9	119.2
3	2	2	4	4	1	Closed	Line	NO	-148.7	-47.6	156.1
4	3	3	4	4	1	Closed	Line	NO	-117.4	-37.1	123.1

	Number of Bus	Name of Bus	ID	Status	Gen MW	Gen Mvar	Min MW	Max MW	AGC	AVR	RegBus Num	Set Volt
1	1	slack bus	1	Closed	155.48	165.35	0.00	1000.00	YES	YES	1	1.00000
2	4	4	1	Closed	350.00	134.23	0.00	1000.00	YES	YES	4	1.00000

	From Number	From Name	To Number	To Name	Circuit	Status	Branch Device Type	Xfrmr	MW From	Mvar From	MVA From
1	1	slack bus	2	2	1	Closed	Line	NO	21.7	49.5	54.1
2	1	slack bus	3	3	1	Closed	Line	NO	83.8	84.9	119.2
3	2	2	4	4	1	Closed	Line	NO	-148.7	-47.6	156.1
4	3	3	4	4	1	Closed	Line	NO	-117.4	-37.1	123.1

Figure 2.4

The top two screenshots are, from top to bottom the Newton-Raphson Generator Data and the Branch State Data. The bottom two are the Gauss-Seidel Generator and Branch State Data. The values that PowerWorld generated for the two different methods are similar, but some small variance does exist between the two methods.

Part 3:

In part three, the four-bus system was analyzed again, this time with Matlab - Matpower.

After downloading the sample case (‘case4gs’) and updating the values to match the lab values, then running the commands

```
>> results = runpf (case4gs, mpoption ('pf.alg', 'NR', 'pf.tol', 1e-6),
'projectone_NR');
```

```
>> results = runpf(case4gs,mpoption('pf.alg', 'GS', 'pf.tol',1e-6),
'projectone_GS');
```

the following results were obtained:

```
projectone_NR

Converged in 0.00 seconds
===============================================================================
|     System Summary
|
===============================================================================

How many?                How much?              P (MW)            Q (MVAr)
---------------------    -------------------  -------------  -----------------
Buses              4     Total Gen Capacity     350.0        -200.0 to 200.0
Generators         2     On-line Capacity       350.0        -200.0 to 200.0
Committed Gens     2     Generation (actual)    505.3             298.2
Loads              4     Load                   500.0             309.9
  Fixed            4       Fixed                500.0             309.9
  Dispatchable     0       Dispatchable          -0.0 of -0.0      -0.0
Shunts             0     Shunt (inj)             -0.0               0.0
Branches           4     Losses (I^2 * Z)         5.26             26.30
Transformers       0     Branch Charging (inj)     -               38.0
Inter-ties         0     Total Inter-tie Flow     0.0               0.0
Areas              1

                          Minimum                      Maximum
                 -------------------------  --------------------------------
Voltage Magnitude   0.969 p.u. @ bus 3          1.020 p.u. @ bus 4
Voltage Angle      -1.54 deg   @ bus 3          2.42 deg   @ bus 4
P Losses (I^2*R)             -                  2.21 MW    @ line 3-4
Q Losses (I^2*X)             -                 11.07 MVAr  @ line 3-4

===============================================================================
|     Bus Data
|
===============================================================================
 Bus      Voltage          Generation             Load
  #   Mag(pu) Ang(deg)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
----- ------- --------  --------  --------  --------  --------
    1  1.000    0.000*   155.26    120.77     50.00     30.99
    2  0.982   -0.458       -         -      170.00    105.35
    3  0.969   -1.539       -         -      200.00    123.94
    4  1.020    2.417    350.00    177.42     80.00     49.58
                        --------  --------  --------  --------
               Total:    505.26    298.19    500.00    309.86
```

```
==============================================================================
|     Branch Data
|
==============================================================================
Brnch   From   To    From Bus Injection   To Bus Injection     Loss (I^2 * Z)
  #     Bus    Bus    P (MW)   Q (MVAr)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
-----  -----  -----  --------  --------  --------  --------  --------  --------
   1      1      2     21.74     25.74    -21.60    -35.09     0.144      0.72
   2      1      3     83.52     64.04    -82.65    -67.24     0.862      4.31
   3      2      4   -148.40    -70.26    150.44     72.69     2.039     10.20
   4      3      4   -117.35    -56.70    119.56     55.16     2.214     11.07
                                                             --------  --------
                                                    Total:     5.260     26.30

projectone_GS

Converged in 0.00 seconds
==============================================================================
|     System Summary
|
==============================================================================

How many?                How much?              P (MW)            Q (MVAr)
---------------------    -------------------  -------------  -----------------
Buses              4     Total Gen Capacity     350.0        -200.0 to 200.0
Generators         2     On-line Capacity       350.0        -200.0 to 200.0
Committed Gens     2     Generation (actual)    505.3             298.2
Loads              4     Load                   500.0             309.9
  Fixed            4       Fixed                500.0             309.9
  Dispatchable     0       Dispatchable          -0.0 of -0.0      -0.0
Shunts             0     Shunt (inj)             -0.0               0.0
Branches           4     Losses (I^2 * Z)         5.26             26.30
Transformers       0     Branch Charging (inj)     -               38.0
Inter-ties         0     Total Inter-tie Flow     0.0               0.0
Areas              1

                          Minimum                      Maximum
                 -------------------------  --------------------------------
Voltage Magnitude   0.969 p.u. @ bus 3          1.020 p.u. @ bus 4
Voltage Angle      -1.54 deg   @ bus 3          2.42 deg   @ bus 4
P Losses (I^2*R)             -                  2.21 MW    @ line 3-4
Q Losses (I^2*X)             -                 11.07 MVAr  @ line 3-4

==============================================================================
|     Bus Data
|
==============================================================================
```

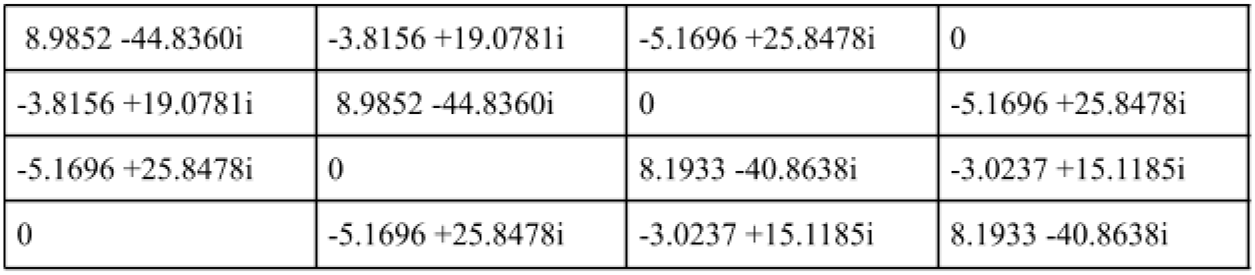
```
 Bus      Voltage          Generation             Load
  #   Mag(pu) Ang(deg)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
----- ------- --------  --------  --------  --------  --------
    1  1.000    0.000*   155.26    120.77     50.00     30.99
    2  0.982   -0.458       -         -      170.00    105.35
    3  0.969   -1.539       -         -      200.00    123.94
    4  1.020    2.417    350.00    177.42     80.00     49.58
                        --------  --------  --------  --------
               Total:    505.26    298.19    500.00    309.86

===============================================================================
|     Branch Data
|
===============================================================================
Brnch   From   To    From Bus Injection   To Bus Injection     Loss (I^2 * Z)
  #     Bus    Bus    P (MW)   Q (MVAr)   P (MW)   Q (MVAr)   P (MW)   Q (MVAr)
-----  -----  -----  --------  --------  --------  --------  --------  --------
   1      1      2     21.74     25.74    -21.60    -35.09     0.144      0.72
   2      1      3     83.52     64.04    -82.65    -67.24     0.862      4.31
   3      2      4   -148.40    -70.26    150.44     72.69     2.039     10.20
   4      3      4   -117.35    -56.70    119.56     55.16     2.214     11.07
                                                             --------  --------
                                                    Total:     5.260     26.30
```

Using the Newton-Raphson method required only three iterations, whereas the Gauss-Seidel method required twenty two iterations! Though the Newton-Raphson method is more complex, and thus would be more complex to write a program for, it is faster. Out of curiosity, running

```
>> [YBUS, YF, YT] = makeYbus(case4gs);
```

Yielded the matrix in Figure 3.1. This Y bus is the same as the PowerWorld and hand calculated Ybuses, but with more precision.



8.9852 -44.8360i	-3.8156 +19.0781i	-5.1696 +25.8478i	0
-3.8156 +19.0781i	8.9852 -44.8360i	0	-5.1696 +25.8478i
-5.1696 +25.8478i	0	8.1933 -40.8638i	-3.0237 +15.1185i
0	-5.1696 +25.8478i	-3.0237 +15.1185i	8.1933 -40.8638i

Figure 3.1

In conclusion, the results from MatPower, PowerWorld, and hand calculations were not the same. Between the two methods of solving, Gauss-Seidel and Newton-Raphson, mathematical differences of the processes would cause slight differences. Between the different programs used, one thing that could impact the results is rounding, and another possible influence would be number of iterations. For example, my calculation by hand was only one iteration, and I also had to round a lot because I couldn’t keep track of so many numbers on my paper or in my head very well. As far as the PowerWorld vs. MatLab Matpower programs, they may round differently, also potentially have different tolerances set. I am not sure how many iterations PowerWorld went through, but it is possible the number of iterations was not the same as the Matpower implementations of the algorithms.

Even on an extremely small 4-bus system, though the results were not astronomically different, there were noticeable variations between methods. However, I do not believe this means that power flow studies are not reliable or not useful; it only means that when performing them, it is important to keep in mind that the result is an approximation and to use the information from them appropriately.